(8 pages) **Reg. No. :** ....

Code No. : 30592 E **Sub. Code : SEMA 6 E**

B.Sc. (CBCS) DEGREE EXAMINATION,
APRIL 2020.

Sixth Semester

Mathematics – Main

Major Elective – IV — CODING THEORY

(For those who joined in July 2017 onwards)

Time : Three hours Maximum : 75 marks

SECTION A — ($10 \times 1 = 10$ marks)

Answer ALL questions.

Choose the correct answer:

1. The total number of words of length n is ————.

(a) n (b) 2n

(c) 2^{n-1} (d) 2^n

2. The maximum number of code words of length $n = 4$ in a code in which any single error can be detected is ————————.

(a) 6 (b) 8

(c) 12 (d) 16

3. The distance of a linear code is the weight of the non zero codeword of ———————— length.

(a) non zero (b) most

(c) least (d) zero

4. The number of different bases for k^2 is ————.

(a) 1 (b) 3

(c) 2 (d) 4

5. The equivalent code of $c = \{000, 100, 001, 101\}$ is ————————.

(a) $\{000, 100, 010, 110\}$

(b) $\{001, 100, 010, 110\}$

(c) $\{000, 010, 001, 110\}$

(d) $\{001, 010, 100, 110\}$

6. The distance of a linear code is the ———————— weight of any non zero code word.

(a) half of (b) two times

(c) maximum (d) minimum

7. The upper bound for the dimension 3 of a linear code of length 6 is ————.

(a) 4 (b) 2

(c) 8 (d) 6

8. The distance of the extended Golay code C_{24} is ————.

(a) 2 (b) 4

(c) 6 (d) 8

9. If $f(x)=1+x+x^3+x^4$ and $g(x)=1+x^2+x^4$ be the polynomials in $k[x]$, then $f(x)+g(x)=$ ————.

(a) $x^2+x^3+x^4$ (b) $1+x+x^2$

(c) $x+x^2+x^4$ (d) $x+x^2+x^3$

10. The cycle shift of the word $U=10110$ is ————.

(a) 0 1 1 0 1 (b) 0 1 0 1 1

(c) 1 0 1 0 1 (d) 1 1 0 1 0

SECTION B — (5 × 5 = 25 marks)

Answer ALL questions, choosing either (a) or (b).

Each answer should not exceed 250 words.

11. (a) Let C be the code of all words of length 3. Determine which code word was most likely sent if 0 0 1 is received. Add a parity check digit to the code words of C.

Or

(b) Calculate $\phi_{0.97}(v,w)$ for each of the following pairs of v and w:

(i) $v = 01101101,\ w = 1000110$

(ii) $v = 00101,\ w = 11010$

(iii) $v = 10110,\ w = 01001$

12. (a) Prove that a linear code of dimension k contains precisely 2^k code words.

Or

(b) If G is a generator matrix for a linear code C of length n and dimension k, then prove that $v = uG$ ranges over all 2^k words in C as n ranges over all 2^k words of length k.

13. (a) Let C be a linear code with parity-check matrix.

$$H = \begin{bmatrix} 1 & 1 \\ 1 & 1 \\ 0 & 1 \\ 1 & 0 \\ 0 & 1 \end{bmatrix}.$$ Find

(i) a generator matrix for $C^{\perp}$,

(ii) a generator matrix for C.

Or

(b) List all the cosets of the linear code $C = \{0000, 1011, 0101, 1110\}$.

14. (a) Can there exist perfect codes for the values $n = 23$ and $d = 7$.

Or

(b) Find generating and parity-check matrices of an extended Hamming code for a linear code with generator matrix.

$$G = \begin{bmatrix} 1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 1 & 1 \end{bmatrix}.$$

15. (a) For a cyclic code $C = \{0000, 1010, 0101, 1111\}$, find the generator polynomial $g(x)$ and then represent each word as a multiple of $g(x)$.

Or

(b) Find a basis and generating matrix for the linear cyclic code of length $n = 7$ with generator polynomial $g(x) = 1 = x + x^3$.

SECTION C — (5 × 8 = 40 marks)

Answer ALL questions, choosing either (a) or (b).

Each answer should not exceed 600 words.

16. (a) Explain why a channel with $p = 0$ is unintervesting.

Or

(b) Let C be the code of all words of length 3. Add a parity-check digit to the code words in C and use the resulting code to answer the following questions.

(i) If 1101 is received can we detect an error?

(ii) If 1101 is received what code words were most likely to have been transmitted?

(iii) If any word of length 4 that is not in the code, closed to a unique code word?

17. (a) Develop the algorithms to find bases for a linear code and its dual.

Or

(b) Find a generator matrix for the linear code generated by each of the following sets. Give the parameters (n, k, d) for each code.

(i) $S = \{11111111,\ 11110000, 11001100, 10101010\}$

(ii) $S = \{100100100,\ 010010010, 001001001, 111111111\}$

(iii) $S = \{101101, 011010, 110111, 000111, 110000\}$.

18. (a) If C is a linear code of length n and dimension k with generator matrix G in standard form, then prove that the first k-digits in the code word $v = uG$ form the word u in K^k.

Or

(b) For each of the following codes, use SDA to decode the given received words.

(i) $C = \{0000, 1001, 0101, 1100$

(1) $w = 1110$, (2) $w = 1001$, (3) $w = 0101$

(ii) $C = \{111000, 001110, 100011\}$

(1) $w = 101010$, (2) $w = 011110$, (3) $w = 011001$

19. (a) What is a lower and an upper bound on the size or the dimension k of a code with $n = 9$ and $d = 57$?

Or

(b) List seven important facts about the extended Golay code C_{24} with generator matrix $G = [I, B]$.

20. (a) $g(x) = 1 + x^4 + x^6 + x^7 + x^8$ generates a 2 error-correctly linear cyclic code C of length 15. Use decoding linear cyclic codes algorithm, decode the received word $w = 110011100111000$ that were encoded using C.

Or

(b) Prove that every cyclic code contains a unique idempotent polynomial which generates the code.
